ET Enterprises electron tubes

technical reprint

R/P080

a monte carlo simulation of photomultiplier resolution

a Monte Carlo simulation of photomultiplier resolution

A G Wright, Electron Tubes Limited, Bury Street, Ruislip, Middlesex HA4 7TA, UK

technical reprint R/P080

abstract

A Monte Carlo simulation of dynode statistics has been used to generate multiphotoelectron distributions to compare with actual photomultiplier resolution results.

In place of Poisson or Polya statistics, in this novel approach, the basis for the simulation is an experimentally determined single electron response. The relevance of this method to the study of intrinsic line widths of scintillators will be discussed.

1 introduction

An understanding of the intrinsic resolution capability of a photomultiplier is fundamental to many applications in scintillation counting and low light level measurements where optimum performance is required. The literature contains many Monte Carlo simulations[1,2] for NaI(Tl) phoswhich and liquid scintillator detectors. A common feature of these studies is their inadequate or even complete lack of photomultiplier simulation.

It was appreciated in the very early days of scintillation spectrometry that multiplication statistics make a significant contribution to resolution. On the assumption that secondary electron emission at an individual dynode is described by the Poisson distribution, Breitenberger[3] derived an expression for the noise factor of an ideal multiplier. Lombard and Martin[4] computed statistical distributions for cascades initiated by single electrons, but found that their distributions were inconsistent with experimentally observed single electron distributions. Prescott and Takhar[5] were able to obtain good agreement between measured and computed multiphotoelectron distributions for a specially focused photomultiplier with an exponential single electron distribution.

Prescott[6] proposed a model for secondary emission based on the Polya distribution which contains as a special case the exponential and Poisson distributions. This was an attempt to allow for the non-uniformity of gain across dynodes. Prescott's well known distribution formula with parameters b and *m* can be fitted to a limited range of experimental distributions. To date there is no universal statistical model that satisfactorily describes all photomultiplier single electron distributions, such as those shown in **figure 1**.

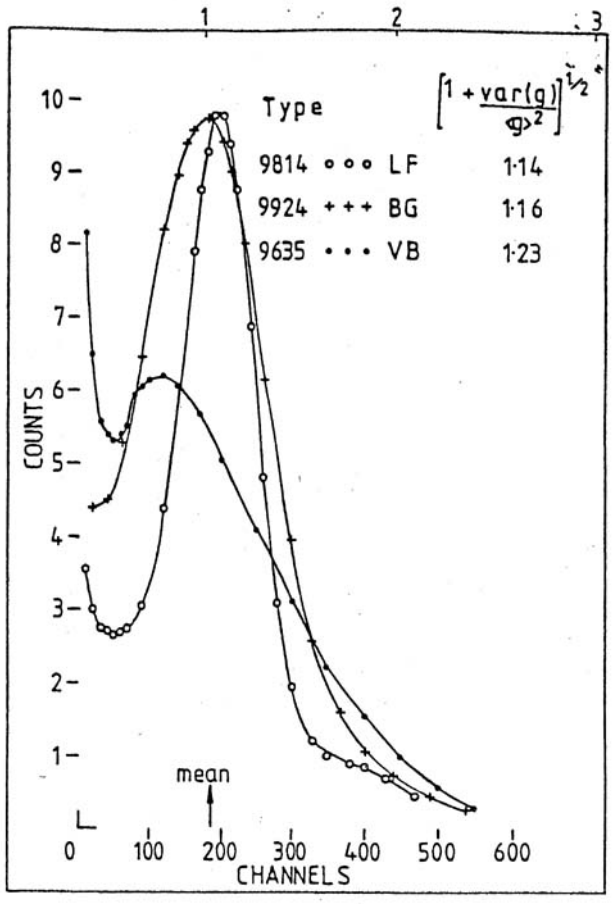

figure 1 single electron pulse height distributions for three types of multiplying structures. The abscissae is arbitrary, but can be related to gain, <*g*>, *Q*, or photoelectrons equivalent.

In the present work a completely different approach will be adopted in which any single electron response (SER) distribution can be used for simulation. The starting point, therefore, will be an actual measured distribution, from which pulse height distributions for multielectron excitations will be generated and compared with experiment.

2 method

multiphotoelectron pulses

For light sources of interest in the majority of photomultiplier applications, for example scintillation light and LED signals, the appropriate statistics are Poisson[7,8]. The combined statistics of photoemission have been shown[8] to be Poisson with a mean value equal to the product of the quantum

efficiency, η, and the average number of incident photons α. The probability of generating *n* photoelectrons is

$$P(\eta) = (\eta\alpha)\eta \exp(\eta\alpha)/\eta! \quad \ldots(1)$$

where $\overline{m} = \eta\alpha$

If the basis for experimental and theoretical computation is taken as photoelectrons, then a knowledge of the quantum efficiency of the photocathode is not required and the question of collection efficiency, a parameter which is not reliably quantified[9], is irrelevant in this instance.

The relative variance for the anode pulse, after amplification through a multiplier characterized by var(g), is obtained by combining the variances in the usual way

$$Var(Q)/\langle Q\rangle^2 = 1/\overline{m}.(1 + var(g)/\langle g\rangle^2) \quad \ldots(2)$$

Where $var(g)/\langle g\rangle^2$ is determined by arithmetic computation from the SER. The expression $(1 + var(g)/\langle g\rangle 2)^{½}$ is referred to as the multiplier noise factor – it is the factor by which the resolution is degraded by the multiplier.

determination of $\overline{m}$

Under single photon excitation, the anode current, I_e, and the total number of counts, n_e, are related to the average multiplier gain $\langle g\rangle$ by[10]

$$I_e = n_e e\langle g\rangle \quad \ldots(3)$$

Where *e* is the electronic charge.

For a light source producing m photoelectrons per pulse at a rate of N per second

$$\overline{m} = I_{\overline{m}}/Neg \quad \ldots(4)$$

Having determined $\langle g\rangle$ from **equation (3)**, **(4)** can be used to determine or set the average number of photoelectrons in the output pulse height distributions.

There is an alternative to the use of the pair of **equations (3)** and **(4)** for determining $\overline{m}$. The multichannel analyzer can be used directly by relating the channel corresponding to m to the mean channel of the SER. The peak of the SER, if one exists, refers of course to the most probably pulse height and only for a well resolved distribution do the mean and the peak concur. For the purpose of photoelectron determination it has become accepted practice to use the fwhm of a distribution to estimate photoelectron yield; this ignores the noise factor and is, therefore, an estimate only.

multiphotoelectron pulse height distributions

For $\overline{m}$<10, for example say 2, the pulse height distribution for a photomultiplier with a good SER is highly structured, first demonstrated by Morton et all,[11] and (2) is not relevant.

To derive an anode output distribution corresponding to m photoelectrons, the following Monte Carlo simulations are executed: n is selected in accordance with the distribution of equation (1); n pulse heights are then selected from the SER distribution. The output pulse height is the sum of the n pulse heights. The process is repeated until a sufficiently well defined distribution is obtain.

If $\overline{m}$ is >10, then the resolution of the anode distribution Q is given by (2).

3 results

The experimental arrangement is straightforward with the photomultiplier under test viewing a pulsed LED. The output is first encoded by a multichannel analyzer and then switched to a picoammeter for the current measurements required for **equations (3)** and **(4)**. The multichannel analyzer was linked to a microcomputer which was programmed for the calculation of standard deviation, noise factor and mean.

For the Monte Carlo simulations a high gain, first stage gain tube was selected, which provided an SER with a well resolved peak. This is illustrated in **figure 2**, together with measured and simulated multiphotoelectron pulse height distributions. The agreement between the measured distributions and the simulation is excellent. Equally good agreement was found for the 9635 type photomultiplier from **figure 1**, which hardly resolves the single electron peak.

Equation (2) was tested over the range $10^1<\overline{m}<10^6$ and results are presented in **figure 3** for a 9814 type photomultiplier with a noise factor of 1.26. The resolution obtained with the LED flasher agrees with the predictions of equation (2) up to photoelectron pulse sizes 10^5. For $\overline{m}>10^5$ the resolution appears to become asymptotic about a value of about ½ %. The origin of this is not entirely clear but it does appear to be an intrinsic photomultiplier effect. A set of ten photomultipliers was tested in the manner of **figure 3**, with similar results, except in the region $\overline{m}>10^4$; in the worst case the deviation

from ideal resolution started at $\overline{m} \simeq 2 \times 10^5$. This range in deviation from ideal resolution can be taken to show that each photomultiplier has a characteristic limiting resolution. In general a contribution from high voltage drift and ripple is possible; as the gain voltage dependency is of the order of 1% per volt, this requires the ripple and any voltage instability to be <100mV. In the present experiment these requirements were met.

Various NaI(Tl) crystals were coupled to the same photomultiplier used to obtain the results in **figure 3**. The resolution for four isotopes, ranging from ^{55}Fe to ^{137}Cs, were measured and the results entered at the appropriate photoelectron positions. We note that the ^{55}Fe and ^{129}I resolutions, which were taken with a thin cleaved X-ray crystal are 5-10 % higher than ideal whereas the ^{137}Cs results is a factor of two higher than the equivalent LED result. The intrinsic resolution of NaI(Tl) is well known and for energies in excess of $\simeq$ 5 MeV other scintillators are sometimes perferred[1]. Apart from this, the effect of photocathode non-uniformity is important at higher energies where the gamma interactions tend to be isotropically distributed throughout the crystal; those interactions that are located near a high spot in cathode sensitivity will obviously produce a bigger signal and vice versa, whereas for ^{55}Fe and ^{129}I the interactions are localized directly below the source.

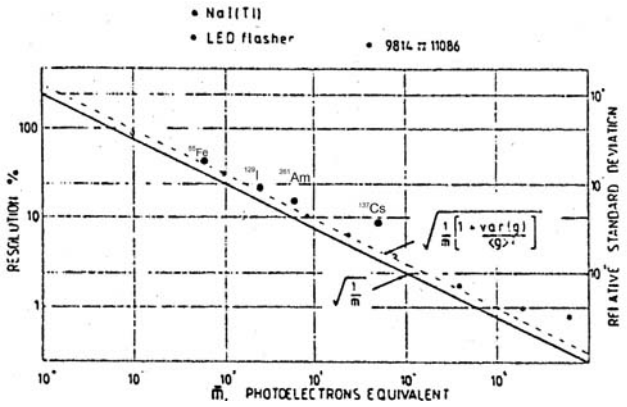

figure 3 resolution as a function of photoelectron number. Resolution is taken as 2.36 x the relative standard deviation.

4 conclusions

In the present investigation the resolution capability of a range of photomultipliers has been accurately predicted from the single electron response noise function together with the $m^{-½}$ dependence from the photoelectron statistics. In this investigation the photocathode spatial illumination was constant. For a proper simulation of scintillation counter behaviour allowance for the non-uniformity of the photocathode response should be made. Although characterising the non-uniformity of a photocathode is straightforward, this cannot be accounted for simply by the addition of another term in equation 2 because in a scintillation counter the distribution on the photocathode will not be uniform but will vary for each event. This requires a Monte Carlo simulation for the light incident onto the specified photocathode. The extension of this work to include photocathode non-uniformity is already under way.

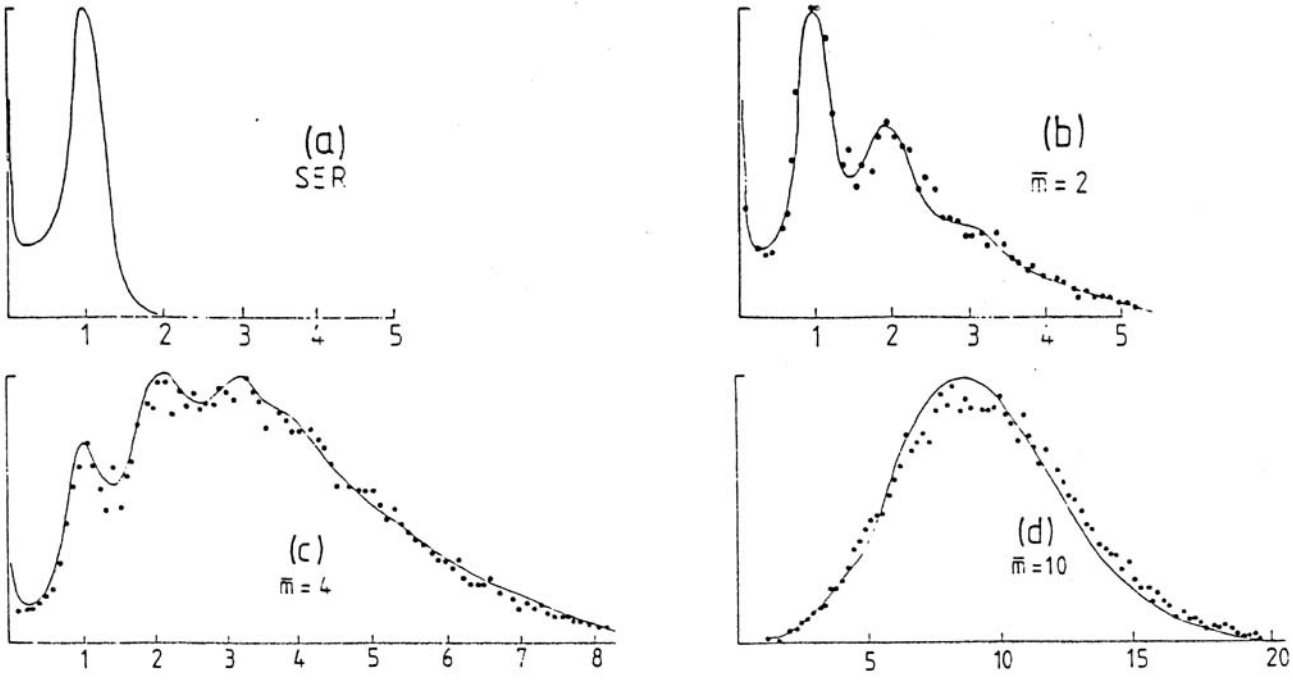

figure 2 multiphotoelectron distributions for a 9829 type photomultiplier –experimental. Monte Carlo simulation generated for (a).

This study highlights the importance of a good single electron response to resolution – the multiplier noise factor in commercial photomultipliers will vary from 1.1 to 1.5, and will degrade the resolution by this factor over the entire energy range. Even a photomulplier with a well resolved SER will not necessarily give good resolution if it has a high proportion of small pulses in the distribution. These undersized pulses contribute very heavily to the variance; this is also true of the high energy region of the SER except that the number of such events is small (see **figure 1**).

references

[1] J J Steyn et al, “Monte Carlo Calculation of Clad NaI(TI) Scinitillation Crystal Response to Gamma Photons”, Nuc. Instr. And Meths. Vol. 107, pp 465-475, 1973

[2] H Klein and H Scholermann, “Improvement of the Light Collection in Scintillation Detectors”, IEEE Trans NS 26, No. 1, pp 373-77, 1979

[3] E Breitenberger, “Scintillation Spectrometer Statistics”, Prog in Nuc. Phys. Vol. 4, pp 56-94, 1955

[4] F J Lombard and F Martin, “statistics of Electron Multiplication”, Rev. Sci. Inst., Vol. 32, No. 2, pp 200-201, 1961

[5] J R Prescott and P S Takhar, “Resoltuion and Line Shape in Scintillation Counters”, IEEE Trans NS 9, No 3, pp 36-45, 1961

[6] J R Prescott, “A Statistical Model for Photomulplier Single Electron Statistics”, Nuc. Inst. And Meths., Vol. 39, pp 173-179, 1966

[7] G K Grau, “Noise in Photoemission Current”, Appl. Optics, Vol. 4, No.1, pp 755-756, 1965

[8] D L Fried, “Noise in Photoemission Current”, Appl. Optics, Vol. 4, No. 1 pp 79080, 1965

[9] A T Young and R E Schild, “Photoelectron Collection Efficiency in Photomultpliers”, Appl. Optics. Vol. 10, pp 1668-72, 1971

[10] A G Wright, “Determination of the Multiplier Gain of a Photomultiplier” J. Phys E: Sci Instrum., Vol. 14, pp 851-5, 1981

[11] G A Morton et al, “Pulse Height Resoltuion of High-Gain First Dynode Photomultipliers”, Appl. Phys. Letters, Vol. 13, p. 356, 1968

[12] E A Bukkum et al, “The Response of BGO Scintillation Detectors to Light Charged Particles”, Nucl. Inst. And Meths., Vol. 225, pp 330-334, 1984

Reprinted with permission, from IEEE Transactions on Nuclear Science, Vol. NS-34, No.1, February 1987

talk to us about your application or choose a product from our literature:

photomultipliers, voltage dividers, signal processing modules, housings and power supplies

ET Enterprises Limited
45 Riverside Way
Uxbridge UB8 2YF
United Kingdom
tel: +44 (0) 1895 200880
fax: +44 (0) 1895 270873
e-mail: sales@et-enterprises.com
web site: www.et-enterprises.com

ADIT Electron Tubes
300 Crane Street
Sweetwater TX 79556 USA
tel: (325) 235 1418
toll free: (800) 521 8382
fax: (325) 235 2872
e-mail: sales@electrontubes.com
web site: www.electrontubes.com

choose accessories for this pmt on our website

an ISO 9001 registered company

ET Enterprises electron tubes

DS_ R/P080 Issue 3 (24/01/11)