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theoretical & practical considerations in the use of photomultipliers for low light level measurements

Theoretical and Practical Considerations in the Use of Photomultipliers for Low Light Level Measurements

by
A. G. Wright,
ET Enterprises Limited

Abstract
The intensity of light falling on a photomultiplier may be estimated by using either a current measuring technique, or by employing the photon counting method. In both cases, photomultiplier noise-in-signal dictates the ultimate accuracy attainable. Where very low light levels are involved, photon counting offers optimum sensitivity.

Considerations relevant to the selection of suitable photomultipliers are: spectral response, dark current, dark count and single electron resolution capability. Guidance on setting up a system for low light level measurements is given.

Ideal Photodetection Statistics
Optical radiation is detected at the photocathode by the emission of photoelectrons following the absorption of photons. The photoelectric effect is a quantum process, governed by Poisson statistics. This means that if M photoelectrons are produced in a time interval T, then the relative fluctuation in M is:

$$\frac{\sigma(M)}{M} = \frac{1}{M^{1/2}} \quad \ldots.(1)$$

The relative variance, by definition $\left[\frac{\sigma(M)}{M}\right]^2$, is:

$$\left[\frac{\sigma(M)}{M}\right]^2 = \frac{1}{M}$$

The signal-to-noise ratio, S/N, is simply:

$$\left[\frac{S}{N}\right]^2 = \left[\frac{M}{\sigma(M)}\right]^2 = M \quad \ldots.(2)$$

Equation (2) sets the lower limit to the accuracy in measuring any signal. Let M refer to a time interval of one second, then the mean cathode current I_k is:

$$I_k = eM \quad \ldots.(3)$$

where e is the electronic charge, and

$$\frac{\sigma(I_k)}{I_k} = \frac{1}{M^{1/2}} \quad \ldots.(4)$$

The more traditional way of specifying the noise in an electrical circuit, where a mean current I_k is defined, is through the shot noise formula. Here time enters via the system bandwidth, Δf.

$$\overline{i_k}^2 = 2e\, I_k\, \Delta f \quad \ldots.(5)$$

where $\overline{i_k}^2$ is the rms fluctuation in the cathode current. The S/N ratio follows from equation (5) and equation (3)

$$\left[\frac{S}{N}\right]^2 = \frac{I_k^2}{\overline{i_k}^2} = \frac{1}{(2\Delta f)} \times M \quad \ldots.(6)$$

Equation (2) states the accuracy attainable by the *photon counting method*, while (5) defines the signal fluctuations inherent in *current measuring techniques*. The two noise formulations, (1) and (5), are reconciled in equation (6) which illustrates their equivalence at a bandwidth of 0.5 Hz. This does not imply that the two signal detection methods offer the same accuracy in practice. To show this, it is necessary to examine the nature of the photomultiplier output, taking account of other sources of fluctuation.

Consider a fictitious photomultiplier which produces a charge q = eg at the anode for each photoelectron leaving the cathode. g is the gain of the multiplier, assumed for the moment not to fluctuate from pulse to pulse. The total charge Q collected at the anode over a time T is $Q = \overline{m}Teg$, where $\overline{m}$ is the mean rate of emission of photoelectrons. The relative variance of Q is given by equation (1) as:

$$\frac{\mathrm{var}(Q)}{\langle Q\rangle^2} = \frac{1}{\overline{m}T} = \frac{1}{M} \quad \ldots.(7)$$

The gain of the multiplier section is a fluctuating process which acts on each photoelectron independently. This arises because of the statistical spread in the secondary emission coefficient of each dynode about a mean value $\overline{\delta}_i$. It can be shown, (1), that equation (7) should be modified to allow for this, as follows:

$$\frac{\mathrm{var}(Q)}{\langle Q\rangle^2} = \frac{1}{\overline{m}T}\left[1 + \frac{\mathrm{var}(g)}{\langle g\rangle^2}\right] \quad \ldots.(8)$$

$$= \frac{1}{\overline{m}T} \quad \frac{\langle g^2\rangle}{\langle g\rangle^2} \quad \ldots.(9)$$

In photon counting we can all but eliminate the effects of varying pulse heights by using a discriminator which produces a *standard pulse* for all photomultiplier pulses above a fixed charge threshold. This implies, in equation (8), that var(g) → o and ideal statistical detection as predicted by equation (1) is achieved.

Returning now to the d.c. detection method. The equivalent circuit of the photomultiplier and associated electronics can be reduced to a parallel combination of resistance and capacitance. The noise bandwidth is simply:

$$\Delta f = 1/4RC \quad \ldots.(10)$$

From equation (6), and including the effect of fluctuating gain, g:

$$\frac{\mathrm{var}(Q)}{\langle Q\rangle^2} = \frac{1}{2\overline{m}\,\tau}\left[1 + \frac{\mathrm{var}(g)}{\langle g\rangle^2}\right] \quad \ldots.(11)$$

Where $\mathcal{T}$ = RC is the integration time constant. Clearly the signal-to-noise ratio can be increased by choosing a long time constant. However, a long time constant can affect the ability of the measuring instrument to follow a time varying output current. In other words, a measurement at time T will be *correlated* to that at T + $\mathcal{T}$ or T − $\mathcal{T}$. It is common in the use of a ratemeter (1), to select a time constant giving a 2% exponential residue, in which case $\mathcal{T}$ = T/4 and equation (11) becomes:

$$\frac{\text{var}(Q)}{<Q>^2} = \frac{2}{\bar{m}T}\left[1 + \frac{\text{var}(g)}{<g>^2}\right] \quad \ldots .(12)$$

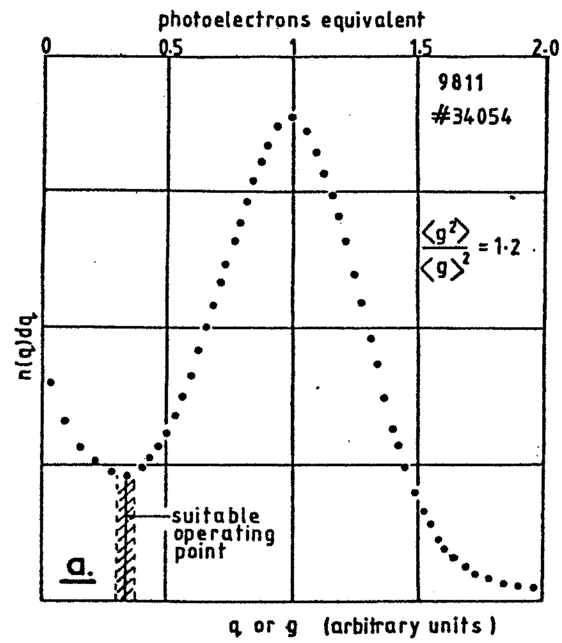
The relative variance is doubled with capacitive integration, (equation (12) when compared with equation (9)), which implies twice the experimental time to achieve the same degree of accuracy. Note further that with capacitive integration, the expression $1 + \text{var}(g)/<g>^2$ degrades the S/N ratio still further.

To summarise: equations (1) and (5) refer to ideal detection of photoelectrons. For low light level measurements it is necessary to amplify the photoelectron signal. The secondary emission process in the photomultiplier is noisy and introduces a factor $1 + \text{var}(g)/<g>^2$ into the expressions for S/N. In the case of photon counting, it is possible to obtain near ideal detection statistics by eliminating the effects of fluctuating gain, but for direct current measurements, this is not possible.

Sources of Non-Ideal Performance

Single Electron Resolution (SER)

The SER of a photomultiplier may be obtained by recording the output pulse height distribution when the cathode is illuminated by a very weak source of light. A light source may be classified as single photon, if the time interval between photoelectrons is much larger than the analysis or dead-time of the recording electronics. In general, a photoelectron rate of $\leqslant 10^4\ s^{-1}$ satisfies this criterion. The distribution of figure 1(a) refers to a linear focused multiplier incorporating a high gain, first stage; 1(b) is the SER for a venetian blind tube. The scale on the abscissa may be variously expressed as photoelectrons equivalent, gain or anode charge.



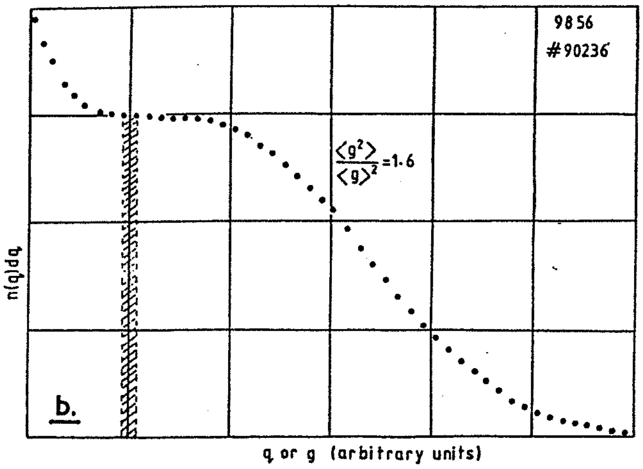


Figure 1 *The SER using a charge-sensitive multichannel analyser a) a linear focused tube with a high gain, first stage, b) a photomultiplier incorporating a venetian blind structure.*

The optimum discriminator threshold for photon counting is a compromise between maximising the count rate and operating in a sufficiently flat region of the distribution. The valley of figure 1(a) is a satisfactory choice, accepting a small loss in counting efficiency. Distributions such as figure 1(b) do not offer such an obvious choice, but operation in the region indicated is usual practice.

It was stated earlier, that for photon counting, the effects of varying pulse height are eliminated by standardising the photomultiplier output as a digital pulse. However, no discriminator is infinitely sharp with regard to its detection threshold: there is always an area of uncertainty either side of the set discrimination level. Pulses falling in the shaded bands of figure 1 are counted with less than 100% efficiency. The shaded area of figure 1(b) clearly represents more noise than the corresponding shaded area of 1(a).

Limitations imposed by Photomultiplier Background

So far we have been concerned with noise-in-signal which is an inescapable manifestation of the quantum nature of radiation and charge. It is essential to distinguish between *noise* and *background* in photomultipliers. Background refers to the measured photomultiplier output in the absence of cathode illumination. Where pulse counting is of concern, then the output count rate and pulse height distribution are of importance. The background current (usually referred to as dark current) is the significant photomultiplier parameter where d.c. detection is under consideration.

It is clear from all the previous theoretical considerations that a photocathode with the highest possible quantum efficiency, η, at the wavelength or wavelength band of interest, should be selected. However, the (S/N) ratio predicted by equations (2) and (6) varies only as $M^{1/2}$ and hence as $\eta^{1/2}$. The dark current and dark counts vary considerably amongst tubes of the same photocathode type. Generally there is an increase in both these parameters, the more red sensitive is the photocathode. The signal/background ratio η/B is undoubtedly a critical selection guide. For a group of similar tubes, η/B will vary more by virtue of B rather than by η; and B is therefore the more relevant parameter in tube selection.

Typical background pulse height distributions for bialkali tubes are shown in figure 2; (a) refers to a linear focused multiplier with a first stage of high gain and (b) represents a venetian blind photomultiplier.

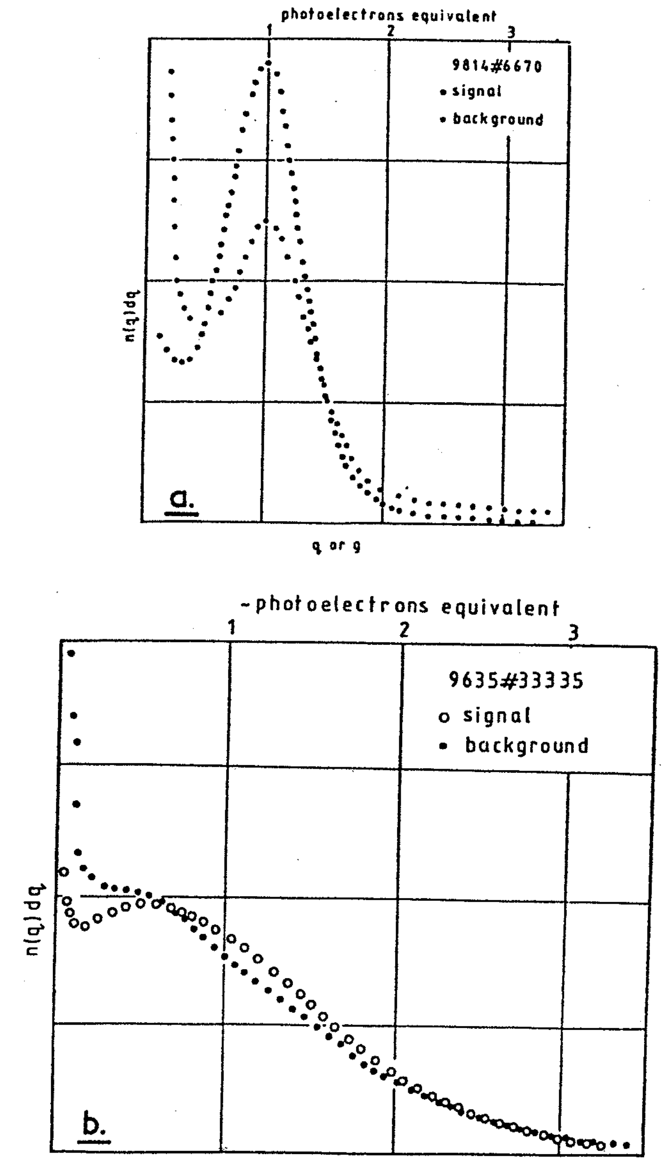


Figure 2 *Signal and background distributions illustrating the excess of small and large pulse height signals in the background. The total number of events in the signal and background spectra is the same, a) linear focused structure b) venetian blind structure.*

Comparing signal and background distributions we note that there are higher proportions of both multiphotoelectron pulses and fractional photoelectron pulses in the background than in the signal. In photon counting, all pulses above threshold are given equal weight; in d.c. measurements, pulses are weighted in proportion to their charge and hence the multiphotoelectron pulses can contribute significantly to the dark current while making a negligible contribution to the dark count rate. With the fractional photoelectron pulses, the situation is reversed and the lower threshold must be chosen with some care.

There is often poor correlation between dark counts and dark current; that is, a tube may have high dark current and yet low count rate or vice-versa. This is partly for reasons already given and partly because dark current has two components:

$$I\,(\text{dark current}) = \int_0^\infty n(q)q\,dq + i \qquad \ldots (13)$$

where i represents the sum of the leakage currents flowing into the anode. Very little is known about the nature of this component of dark current, but there is evidence to suggest that this, together with the cosmic ray component of background, contributes 1/f noise. (To be consistent it should be referred to as 1/f background). A plot of the dark current frequency distribution generally has a form similar to that shown in figure 3. The corner frequency, f_c, is rarely below 1 Hz and may be as much as 1 kHz. The presence of low frequency noise imposes a lower limit to the sampling time T or time constant τ which should both be $<< f_c$.

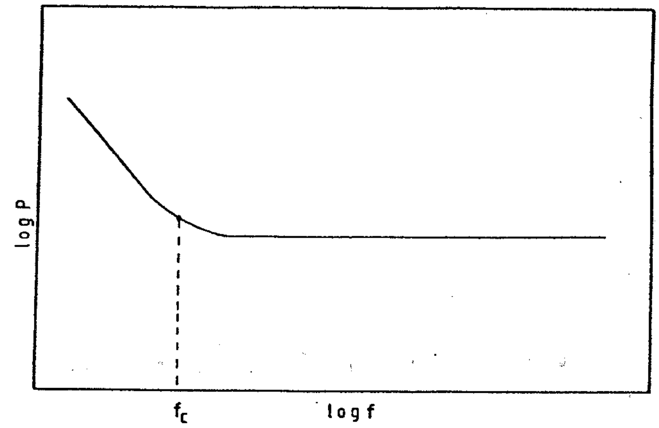


Figure 3 *A dark current frequency distribution showing an excess in the low frequency components compared with white noise.*

Basic Counting System

Low level light measurements require extreme care in the electrical wiring. Electrical and magnetic shielding and good high frequency earthing techniques must be employed. Any material in contact with, or in close proximity to, the photocathode should be maintained at cathode potential. Failure to do so can result in unstable tube operation. With the 'anode earthed' configuration it is advisable to connect the cathode pin to a conducting coating or shield around the cylindrical surface of the tube. For low level photon counting, the earthed cathode configuration offers the best immunity to electrical interference and usually the most stable photomultiplier operation. High and variable count rates can, however, dictate the need for direct coupling between anode and the associated electronic circuits.

Photon counting electronics need not be sophisticated. The basic requirements call for a stable amplifier/discriminator combination capable of handling pulses over a dynamic range of 100:1. The user is faced with a problem in choosing how to proportion the overall gain of the system between the photomultiplier, the amplifier and the discriminator. This is resolved by recording a family of integral count rate curves as shown in figure 4, for a range of sensitivity settings. A plot of S/B will show the range of sensitivities over which acceptable performance obtains. For the photomultiplier/electronics combination shown, it is clear that the sensitivity of the counter must be > 1 mV. The enhanced background, evident at high sensitivity, was due to electrical pick up in this particular system.

There is considerable advantage to be gained by reducing the background rate by cooling. With the S20 type photocathodes, background reduction by a factor

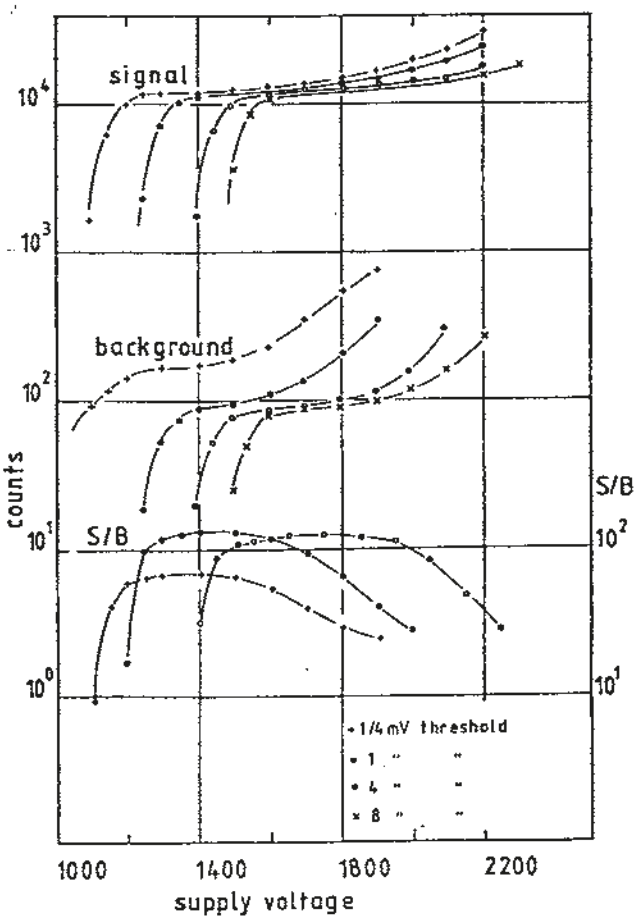


Figure 4 *The use of a family of integral count rate distributions to optimise the performance of a photon counting system.*

of ~100 can be achieved by cooling the tube from 20 °C to −20 °C; the bialkali photocathodes show a reduction by a factor of ~3 over this temperature range.

In those instances where the light can be focused, the use of a photomultiplier with reduced cathode area can obviate the need for cooling. The THORN EMI 9863/9893 range of photon counting tubes, for example, provides the option of a 2.5 mm or 10 mm diameter photocathode.

Range of d.c. and Photon Counting Techniques

Consider the measurement of a source producing 10^3 photons per second incident on a photocathode with $\eta = 10\%$. The sensitivity of a typical, commercially available amplifier/discriminator requires $\langle g \rangle \simeq 10^7$. The mean anode current is thus:

$$\begin{aligned} I_a &= M\,e\,\langle g \rangle \\ &= 10^2 \times 1.6 \times 10^{-19} \times 10^7 \\ &= 1.6 \times 10^{-10}\ \text{A} \end{aligned}$$

Given a bialkali photomultiplier selected for low dark current (<0.5 nA) and dark counts (<100 s^{-1}), then photon counting offers an efficient means of measuring the signal. The same tube, operated at $\langle g \rangle = 10^7$, in the d.c. mode would be required to measure a signal of about one third of the background output which is rather marginal in view of the 1/f limitations already mentioned. The lock-in amplifier technique, employing a light chopper, overcomes the 1/f limitation by successively measuring S + B and B and provides a more accurate answer.

The advantage of photon counting becomes more obvious, the lower the photoelectron rate.

Measuring anode currents below 0.1 nA, corresponding to a photoelectron rate of 10 s^{-1} for the example cited, presents a considerable challenge. The ability to recover a signal, one tenth that of the background, is shown in figure 5 taken from (2). Synchronous detection, that is the interleaving of S + B and B measurements, was used to extract a count rate of 0.043 s^{-1} from a cooled dark count of 0.459 s^{-1}. Although the experiment required several hours of progressively updating S and B estimators, there is *no alternative technique* capable of this degree of signal recovery.

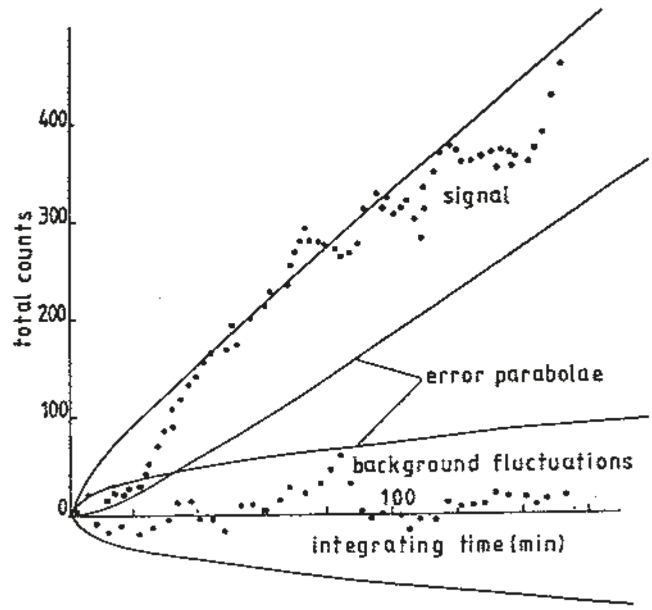


Figure 5 *The technique described in (2) for the measurement of a very feeble light source.*

References

1) R. Jones, C.J. Oliver & E.R. Pike, Applied Optics, 10, No. 7, 1673–80 (1971).

2) C.J. Oliver, Reprint, Fig. 9, R/P 066.

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e-mail: sales@et-enterprises.com
web site: www.et-enterprises.com

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Sweetwater TX 79556 USA
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