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photomultiplier

# determination of the multiplier gain of a photomultiplier

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## abstract

*Two means for measuring multiplier gain are described. The determination of the total number of single electron pulse, N enters into both methods. Use of a multichannel analyser of calibrated input sensitivity reveals that the error in determining  $\langle g \rangle$  arises mainly from an inability to assess whether all contributions from the low energy region of the pulse height distribution have been included in N. It is concluded that mean gain measurements with better than 10% accuracy are unlikely for most photomultipliers. This has serious implications for the absolute measurement of light intensities through the need to know F, the collection efficiency, the determination of which can be sensitive to the assumed values of  $\langle g \rangle$  and var  $\langle g \rangle$ .*

## 1 introduction

The overall gain  $G$  and the multiplier gain  $g$  are fundamental photomultiplier parameters. The mean gain  $\langle G \rangle$  of a photomultiplier is defined here as the ratio of the anode current to the cathode current. The mean, multiplier gain  $\langle g \rangle$ , is the ratio of the mean anode charge, produced by a single photo-electron, to the electronic charge  $e$ . It is important to appreciate that  $\langle G \rangle$  and  $\langle g \rangle$  are not the same. Only a proportion of the emitted photoelectrons reach the first dynode and of these, a fraction fail to propagate. To account for this it is necessary to introduce a parameter,  $F$ , the collection efficiency. The definitions of  $\langle G \rangle$  and  $\langle g \rangle$  imply

$$\langle G \rangle = F \langle g \rangle \quad \dots(1)$$

The signal-to-noise ratio is always an important consideration in any low light level application because this sets the theoretical limit to the accuracy with which a photomultiplier measurement can be made.

The S/N ratio follows directly from the shot noise formula and has the following form

$$\left(\frac{S}{N}\right)_{\text{anode}} = \frac{F \langle I_a \rangle}{2e \langle G \rangle \Delta f} \left[1 + \frac{\text{var}(g)}{\langle g \rangle^2}\right]^{-1/2} \quad \dots(2)$$

where  $\Delta f$  is the equivalent noise bandwidth and  $\langle I_a \rangle$  is the mean anode current. The expression in square brackets represents the noise from the multiplier and is taken from Jones et al (1972).  $F$ ,  $\langle G \rangle$ ,  $\langle g \rangle$  and  $\text{var}(g)$  enter into the S/N calculation which implies that the detailed shape of the pulse height distribution must also be known if  $\text{var}(g)$  is to be computed with any accuracy.

The anode to cathode current ratio method is a standard procedure for determining  $\langle G \rangle$ . However, this invariably gives unreliable results because of the practical difficulties in measuring small cathode currents accurately. Young and Schild (1971) and Coates (1973) have described a means of density filters, which undoubtedly provides a more reliable value for  $\langle G \rangle$ . Robben (1971), Young and Schild (1971) and Klobuchar et al (1974) determined  $\langle g \rangle$  by counting the anode pulses derived from a source of single photons. The corresponding anode current,  $I$  is given by

$$I = \int_0^\infty n(q) q dq \quad \dots(3)$$

where  $n(q)$  is the differential pulse height distribution of the anode pulses.

The average anode charge per photoelectron is

$$\langle q \rangle = \int_0^\infty n(q) q dq / \int_0^\infty n(q) dq \quad \dots(4)$$

$$= e \langle g \rangle \quad \dots(5)$$

$N = \int_0^\infty n(q) dq$  is the total number of anode pulses per second and from (3) and (4)

$$I = \langle g \rangle Ne \quad \dots(6)$$

from which  $\langle g \rangle$  may be calculated.

A criticism that can be levelled at perhaps all of the previous investigations is the lack of serious attempts to assess the experimental errors. The paper by Klobucher et al (1974) is particularly misleading in this respect because they claim that the method is accurate without providing the necessary experimental justification. It will be shown that the error in determining  $\langle g \rangle$  arises mainly from an inability to assess whether all contributions have been included in  $N$ . The uncertainty in  $N$  casts doubt on the validity of the concept of mean multiplier gain, at least in so far as the photomultiplier types investigated here are concerned.

Two methods for determining  $\langle g \rangle$  are described, both of which require a knowledge of  $N$ . The most

probable gain  $\hat{g}$  may be defined for those photomultipliers capable of resolving a single electron peak in the output pulse height distribution. A means, independent of  $N$ , for determining  $\hat{g}$  to within an accuracy of 5% is given. This method is based on the use of a multichannel analyser (MCA) of known charge sensitivity.

## 2 experimental procedure

The multiplier gain measurements reported in the references quoted above all rely on an integral discriminator for the determination of  $N$ . A charge-sensitive, multichannel analyser was used in this investigation, because it can be readily calibrated in

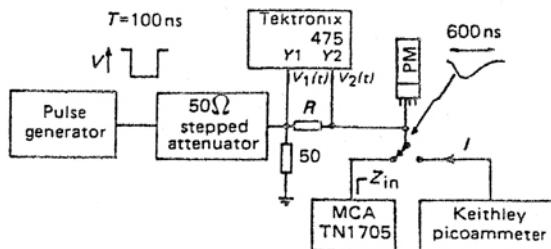


figure 1 the experimental arrangement for determining  $\langle g \rangle$

terms of pC per channel, which then allows an evaluation of  $\langle g \rangle$  from equations (4) and (5).

The analyser (Tracor TN 1705) was calibrated by injecting a known charge into the instrument. The experimental configuration used is shown in figure 1. Provided  $R$  is a pure resistance, then the injected charge is

$$Q = \int_0^T \frac{(v_1(t) - v_2(t))dt}{R} \quad \dots(7)$$

with the symbols as defined in figure 1.  $v_1(t)$  and  $v_2(t)$  are different functions of  $t$  because of the integrating action of the analyser. However choosing  $R \gg |Z_{in}|$  renders  $v_2 \ll v_1$  and reduces (7) to the following convenient form

$$Q = VT/R \quad \dots(8)$$

$R = 100 k\Omega$  was found to be a satisfactory value for calibration. Choosing  $R \ll 100 k\Omega$  invalidates the requirement  $v_2 \ll v_1$  and if  $R$  is too large the contributions from stray and parallel capacitance become significant. The charge sensitivity of the analyser is affected by the total capacitance connected across the input. All coaxial cables and associated instruments were therefore left in situ throughout the experiment. The calibration shown in figure 2 was obtained by stepping through the attenuator settings, noting the corresponding channel positions

registered. The inset shows that the zero intercept adjustment is set properly. The analyser has a lower level discriminator which the manufacturers

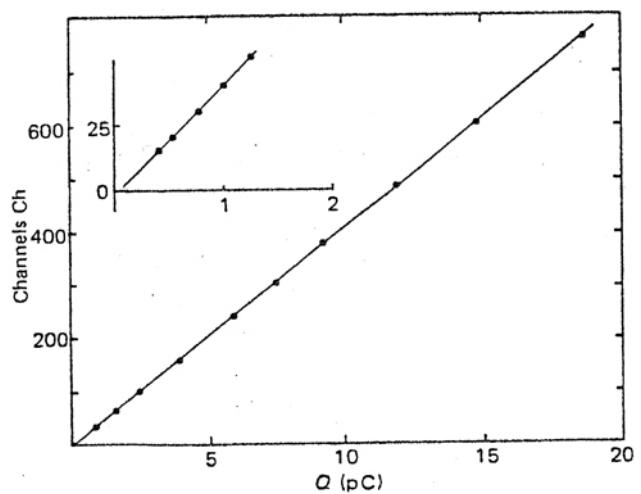


figure 2 calibration curve for the charge sensitivity of the multichannel analyser. The inset shows that the analyser is properly adjusted with regard 'zero intercept'.

recommend should be adjusted to exclude circuit noise. The noise spectrum was recorded without voltage applied to the photomultiplier and observed to be exponential in character. Virtually all circuit noise could be eliminated by setting the lower level discriminator to exclude the first nine out of 1024 channels.

Two photomultipliers were selected for gain determination. The Electron Tubes type 9798 has a box and grid structure with the ability to resolve a single electron peak; the 9635 photomultiplier incorporates a venetian blind multiplier which barely

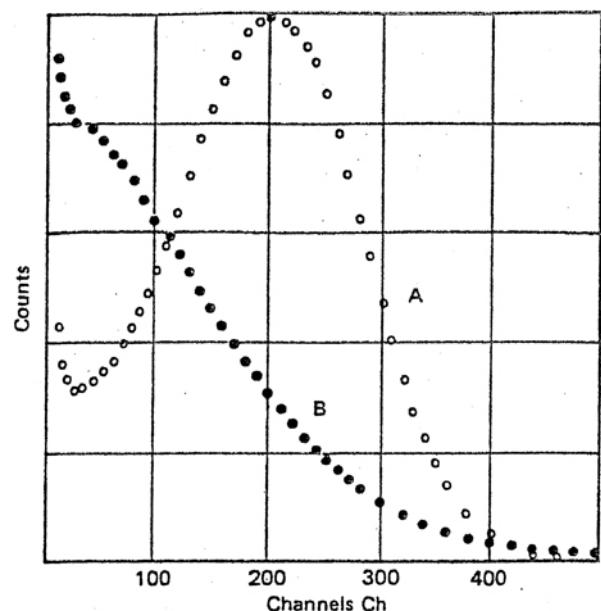
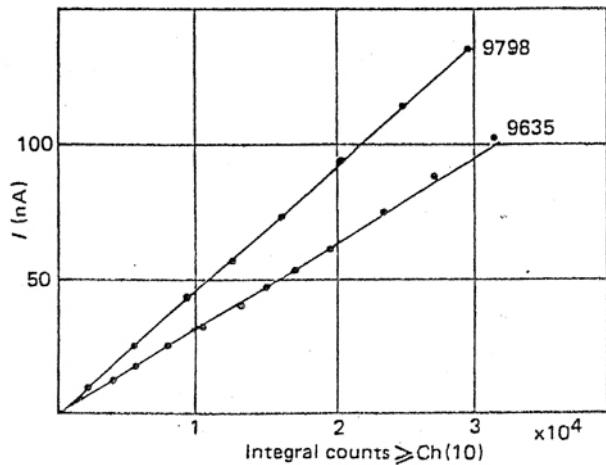


figure 3 single electron distributions for: A, 9798 and B, 9635 type photomultipliers.

resolves single electrons. The pulse height distributions obtained from a very weak, random, source of single photons are shown in **figure 3**. Since the analyser is calibrated, the abscissa may be variously expressed as channels, coulombs or gain,  $g$ . The analyser sensitivity, for the instrument settings used, was  $2.44 \times 10^{-14}$  C/Ch, based on the calibration of **figure 2**. Dividing this by the electronic charge gives the equivalent gain scale of  $1.52 \times 10^5$  per channel.

The intensity of the light source was varied without altering the gain of the photomultiplier. For each light level setting, the integral count rate  $>$  Ch 10 was noted together with a measurement of the anode current  $I$ . The analyser was used in the 'live' mode to allow for dead time. However, count rates were kept below  $3 \times 10^4$  s $^{-1}$  so that the dead time correction was always small. The multiplier gains derived from the slopes of the curves in **figure 4** are given in **table 1**.



**figure 4** the variation of anode current with signal counts.

**table 1** determination of  $\langle g \rangle$

| Tube Type | method 1                   | method 2  | method 3           |
|-----------|----------------------------|---|--------------------|
|           | $\langle g \rangle = I/Ne$ | $\langle g \rangle = \int_{10}^{1023} n(q)qdq/eN \hat{g}$ |                    |
| 9798      | $2.85 \times 10^7$         | $2.94 \times 10^7$  | $3.05 \times 10^7$ |
| 9635      | $1.96 \times 10^7$         | $2.08 \times 10^7$  | —                  |

The alternative method for determining  $\langle g \rangle$ , through numerical evaluation of the integral in the numerator of equation (4), gave the results in column 3 of **table 1**. Using equation (5) and the calibration in **figure 2**, gives the average anode charge  $\langle g \rangle$ , per photoelectron in coulombs or equivalent channels. For the 9798 tube, it is noted that  $\langle g \rangle$  ( $\equiv$  Ch 193) lies very close to Ch 200, the peak of the single electron response.

### 3 discussion

#### 3.1 sources of error

The photomultiplier output pulse height spectrum for the background is different from that for single photon illumination. Background spectra for most photomultipliers contain a higher proportion of both small and multiphotoelectron equivalent pulses than the signal distribution. The assumption is made in equation (5) that all pulses originate as single electrons at the cathode and it was therefore necessary to examine the significance of the background contribution. The integral count rates (0.05 to 5.0 photoelectrons equivalent) were  $150 \text{ s}^{-1}$  and  $1.2 \times 10^3 \text{ s}^{-1}$  for the 9635 and 9798 photomultipliers, respectively. Referring to the 9635 photomultiplier, the background contribution to both  $I$  and  $N$  in **figure 4** is negligible, even after allowing for the different pulse height distribution that applies, and hence no correction was made. The 9798 photomultiplier has a S20 photocathode with low work function. The predominant source of background is likely to be due to the thermionic emission from the photocathode and the pulse height distribution should be the same as that for single photon excitation. This was checked and verified. Background subtraction was unnecessary in this investigation although this would not have presented any practical difficulties.

It was also noticed that the spectrum for a random source of single photons differs from that obtained with a synchronous source of single photons, such as a highly attenuated pulsed light emitting diode. The spectrum for the latter appears to have a smaller proportion of undersized pulses, although this might be the result of some experimental artifact. If this is genuine, however, then the implications are (i) a proportion of the small pulses are signal induced and (ii)  $\langle g \rangle$  for a photomultiplier used for synchronous detection of photons is different from that which applies when the source of light is random. This will be investigated further by using a multi-channel analyzer which incorporates a linear gate.

Following Coates (1973), equation (6) should be modified to allow for the effects of afterpulses, thus

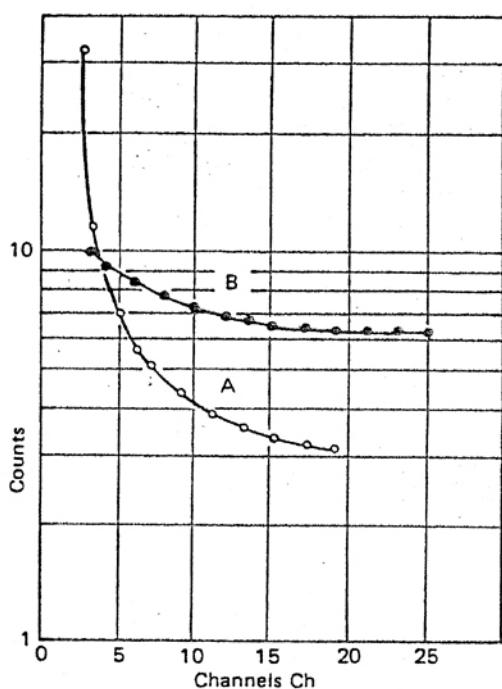
$$I = \langle g \rangle Ne(I + \bar{\gamma}) \quad \dots(9)$$

where  $\gamma$  is the afterpulse per photoelectron and  $\bar{\gamma}$  is the average height of afterpulses in photoelectron equivalent. Coates gives a figure of  $n = 6.2$  for one of the tubes he examined whereas for the multipliers of this investigation  $n$  is likely to be  $\sim 3$  because of the lower  $k - d_1$  voltages used.  $\gamma$  was not meas-

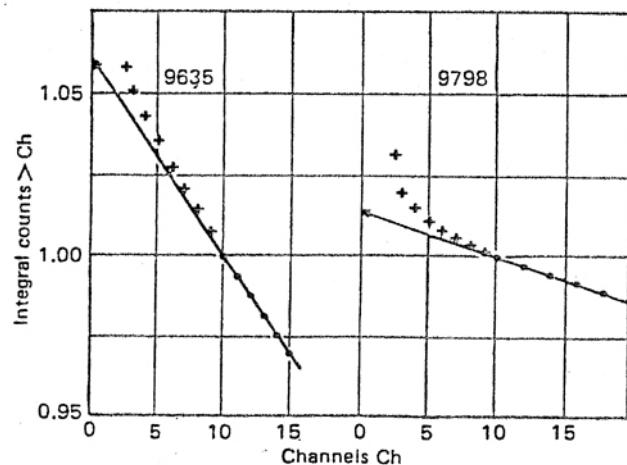
ured for the actual photomultipliers used, but it is known from standard production tests that  $\gamma < 0.01$ .

The y-input calibration of the oscilloscope was checked using a Keithley 150B digital multimeter, and the time-base calibration with a timer counter. Consequently, the determination of the charge sensitivity of the analyser is considered to be correct within 3%. The good agreement between the pairs of readings in **table 1** is therefore not surprising and supports the figure quoted for the calibration accuracy.

The two methods for determining  $\langle g \rangle$  are not independent because equations (4) and (6) are both functions of  $N$  for which a lower cut-off at channel 10 was imposed. The contribution to  $\int n(q)qdq$  from small pulse heights is negligible since  $n(q) q \rightarrow 0$  as  $q \rightarrow 0$ ; but they have exactly the same weight as all other pulses with regard to their contribution to  $\int n(q)qdq$ . An attempt was made to estimate the contribution to  $N$  from pulse heights less than Ch 10. The gain of the photomultiplier was doubled by increasing the supply voltage until the anode current, corresponding to the count rate in **figure 3**, doubled. Channel 10 of the analyser now corresponded to Ch 5 in **figure 3** and by increasing the tube gain by a further factor of 2,  $n(q)$  down to an equivalent channel of 2.5 was determined. The extensions to the pulse height distributions obtained by this method are shown in **figure 5**. It should be noted that the ordinate scale is logarithmic so that the number of small pulses is indeed large. When these additional data are included in the estimation of  $N$ , the rate of increase of  $n(q)dq$  steepens as  $Ch \rightarrow 0$ .



**figure 5** the extension of the single electron distribution to low channel numbers: A, 9798 and B, 9635.



**figure 6** integral pulse height distributions showing contributions from low channel numbers, +, determined from measurements made at higher tube gain. The ordinate has been normalised to the number of counts  $\geq Ch$  10.

When the contributions from the low energy region of the spectrum are known, it might appear that  $N$  can be determined by linear extrapolation of experimental readings. More detailed data show that this is not so and that any extrapolation, linear or otherwise is arbitrary. The mean gain figures quoted in **table 1** need to be reduced by about 10% and the quoted accuracy in  $\langle g \rangle$  is considered to be  $\pm 10\%$  based on **figure 6**.

There is an alternative method for expanding the pulse height distribution. The overall voltage to the photomultiplier is kept constant while the analyser preamplifier gain is increased to reveal more of the low energy region of the spectrum. This method was attempted but found to be unsatisfactory because of excessive amplifier noise. Although the adopted method is open to criticism on the grounds that the operating conditions of the tube are altered, this cannot account for the numbers measured in the low channel region of the distribution.

### 3.2 sources of small pulses

Lombard and Martin (1961) have computed output pulse size distributions for multipliers, based on the assumption that the secondary emission process is Poissonian. They show how the shape of the distribution depends critically on the magnitude of  $\bar{\delta}_1$ , the mean gain of the first stage of multiplication. As  $\bar{\delta}_1$  increases, the distribution becomes more peaked and for  $\bar{\delta}_1 \geq 10$ , the output distribution approaches a Gaussian one. Although many photomultipliers are commercially available with high gain, first stage, all appear to produce an excess of small amplitude pulses. For example, Barton et al (1964) have reported on ten venetian blind type photomultiplier, none of which produced a discernible single electron peak. Linear focused photo-

multiplier incorporating a GaP, first dynode ( $\bar{\delta}_1 \approx 30$ ) provide a well resolved peak but in the presence of an excess of undersized pulses (see for example, Morton et al (1968), Coates (1970, 1971)). The present author has examined many hundreds of photomultipliers, of various types and from most manufacturers, and always found results consistent with those already quoted. Prescott (1966) suggested that  $\delta$  varies across the surface of the dynode and postulated the Polya distribution as a means for describing output pulse height distributions. The secondary emission coefficient is likely to be non-uniform because of the manufacturing processes used for laying down the secondary emitting surfaces. In addition, it is certainly true that the collection efficiency between the first and second dynodes is dependent upon the point of impact on the first dynode. Coates (1970) points out that 'electrons in the cascade are lost if they strike the dynode supporting structure, an insensitive area of the dynode, or if they miss the dynode completely'. The last mentioned loss mechanism clearly applies to electrons moving parallel to the slat planes of a venetian blind dynode, for example. Oliver and Pike (1970) and Coates (1973) suggest that elastically scattered and back diffused primary electrons also contribute to the low pulse height region of the spectrum.

Weldon et al (1979) have done Monte Carlo simulations of secondary emission from alkali halides. The predicted number distribution is non-Poissonian, in good agreement with their experimental observations which also show an excess in the low secondary number region. They suggest that their results could be more generally applied to explain the single electron pulse height distributions observed in photomultipliers.

It was observed that the shape of the single electron response was sensitive to the wavelength of the incident light, particularly for the 9798 photomultiplier. This was not a peculiarity of the particular 9798 chosen but applies quite generally to tubes of this type. The spectrum was wavelength independent for  $\lambda \geq 550$  nm but with  $\lambda = 420$  nm, for example, the distribution was distinctly broader. The most likely explanation lies in the relatively simple electron optics of this tube design. The extraction field at the cathode is very weak and consequently photoelectrons produced by blue light ( $hc/\lambda \sim 3$  eV) do not focus on to  $d_1$  in the same way as do photoelectrons with less initial energy. Moszynski and Vacher (1977), have shown that the transit time dispersion is wavelength dependent, thereby supporting the aforementioned. The situation is further complicated by the fact that the inner surface of the window, the inner wall of the glass envelope

between the window and  $d_1$  and the first dynode itself are all photosensitive. The transmission coefficients for S11, S20 and bialkali photocathodes vary almost linearly with  $1/\lambda$ , being  $\sim 0.1$  in the UV and  $\sim 0.8$  in the infrared region of the spectrum (Timan 1976). This means that a proportion of the photocurrent is always derived from surfaces other than the photocathode layer on the window and the contribution will be strongly wavelength dependent. Tube manufacturers undoubtedly design the electron optics of photomultipliers with a view to optimizing the collection for the window and one can only speculate as to where the photoelectrons from the other surfaces land.

### 3.3 derivation of a gain-voltage characteristic

The analyser used for this investigation had a sensitivity of  $2.44 \times 10^{-14}$  C, which is high by industry standards. The range over which  $\langle g \rangle$  can in general be measured is likely to be  $5 \times 10^6$  to about  $10^8$ . The sensitivities of most commercially available multichannel analysers do not permit accurate measurements below  $5 \times 10^6$  while tubes of the type tested here do not always operate reliably for  $\langle g \rangle$  in excess of  $10^8$ . However, a complete gain voltage characteristic can always be derived from a single measurement of  $\langle g \rangle$ , by noting the variation of  $I$  with operating voltage,  $V$ , starting at  $V(\langle g \rangle)$ .

## conclusions

The accuracy with which the mean gain of a multiplier can be determined is governed principally by the shape of the low energy region of the differential pulse height distribution. This work shows that linear extrapolation of an integral count rate in an attempt to include all low energy pulses leads to an underestimate of the true number. Detailed information on the low energy region of the spectrum still does not ensure a reliable estimate of either  $\langle g \rangle$  or of the error in  $\langle g \rangle$ . Photomultipliers with a well-resolved single electron response can be characterized by  $\hat{g}$  the most probable gain, but for the majority of commercial photomultipliers,  $\hat{g}$  is not defined.

The mean gain of a multiplier is not uniquely defined by the applied voltage distribution. Other factors which need careful consideration are the following:

- the wavelength of the light;
- the degree of afterpulsing exhibited by the photomultiplier;
- the area of illumination of the photocathode.

It is only possible to quantify light signals in absolute units if the fundamental parameters  $\langle G \rangle$ ,  $\langle g \rangle$ ,  $F$  and the quantum efficiency are known. Equations (1) or (2) have already been used by Coates (1973), Young and Schild (1971), Lakes and Poultney (1971) and Foord et al (1969) to determine  $F$  and in view of the findings of this work, it would seem that the uncertainty in  $F$  may be larger than these workers have acknowledged.

## references

- [1] Barton J C, Barnaby CF and Jasain B M 1964 An investigation of noise in venetian blind photomultipliers. *J. Sci. Instrum.* 41 599-604
- [2] Coates P B 1970 The edge effects in electron multiplier statistics. *J. Phys D: Appl. Phys.* 3 1290-6
- [3] Coates P b 1971 Noise sources in the C31000D photomultiplier. *J. Phys. E:Sci. Instrum.* 4 201-7
- [4] Coates P B 1973 Photomultiplier collection efficiencies and non-Poissonian pulse height distributions *J. Phys.D: Appl. Phys.* 6 153-63
- [5] Foord R, Jones R, Oliver C J and Pike E R 1969 The use of photomultiplier tubes for photon counting *Appl. Opt.* 8 1975-89
- [6] Jones R, Oliver C J and Pike E R 1971 Experimental and theoretical comparison of photon-counting and current measurements of light -intensity *Appl. Opt.* 10 163-80
- [7] Klobuchar R L, Ahumada J J, Michael J V and Karol P J An accurate method of photomultiplier gain determination *Rev. Sci. Instrum* 45 1071-6
- [8] Lakes R S and Poultney S K 1971 Direct measurement of the quantum counting efficiency of RCA C31000E/F photomultipliers at 7328A *Appl. Opt.* 10 797-800
- [9] Lombard F J and Martin F 1961 Statistics of electron multiplication *Rev. Sci. Instrum* 32 200-1
- [10] Morton G A, Smith H M and Krall HR 1968 Pulse height reolution of high gain first dynode photomultipliers *Appl. Phys. Lett.* 13 356-7
- [11] Moszynski M and Vacher J 1977 Influence of incident light wavelength on time jitter of fast photomultipliers *Nucl. Instrum . Meth.* 141 319-23
- [12] Oliver C J and Pike E R 1970 The edge effect in electron multiplier statistics *J. Phys. D:Appl. Phys.* 3 L73-5
- [13] Prescott J R 1966 A statistical model for photomultiplier single-electron statistics *Nucl. Instrum. Meth.* 39 173-9
- [14] Robben F 1971 Noise in the measurement of light with photomultipliers *Appl. Opt.* 10 776-96
- [15] Timan H 1976 Optical characteristics and constants of high efficiency photoemitters *Rev. Tech. Thomson-CSF* 8 4-84
- [16] Weldon V L, Delaney C F G and McDonald I R 1979 The statistics of secondary electron emission from some alkali halides *IEEE Trans. Nucl. Sci.* NS-26 419-21
- [17] Young A T and Schild R E 1971 Photoelectron collection efficiency in photomultipliers *App. Opt.* 10 1668-72

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